Cambridge Assessment International Education

Cambridge International AS & A Level

THINKING SKILLS **9694/31**

Paper 3 Problem Analysis and Solution **October/November 2022**

2 hours

7884281105

You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
  Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
  In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

DC (CE/CT) 320431

1 Liam runs a small business which provides training courses. He runs courses every Tuesday, Wednesday, Thursday and Friday. The courses take place in meeting rooms that Liam hires from local hotels.

Liam charges $100 per delegate for half-day courses and $200 per delegate for whole-day courses. The costs to Liam for renting rooms are shown in the table below.

Room size	*Maximum number of delegates*	*Cost for half day ($)*	*Cost for whole day ($)*
Small	10	400	750
Medium	25	800	1500
Large	50	1500	2400

Liam must book the room four weeks before the course takes place and then cannot change the booking. If more delegates apply for the course, he will allow them to register, but only if there is space. Each course must take place in one room.

Each Monday, Liam books the rooms that he needs for the courses that will run four weeks later. The courses that need to have rooms booked this week are as follows:

Day	*Times*	*Delegates currently registered*
Tuesday	Morning	15
Wednesday	Afternoon	9
Thursday	All day	20
Friday	Morning	26

(a) Assuming that no further applications are received for these courses, what is the profit that Liam will make if he books the smallest room that is suitable for the number of delegates registered for each course? [2]

Liam has not yet booked the rooms. He is considering how many additional applications for a course would be needed in order for it to be worth booking a larger room than is required for the current number of delegates.

(b) If more delegates apply for the course scheduled for Wednesday afternoon:

(i) How much extra profit could Liam make if he still books the smallest room for this course? [1]

(ii) How many additional delegates would need to apply in order for Liam to make more profit by booking a larger room? [2]

Looking at the courses that he has run before, Liam believes that the number of additional applications for a course will be at least one quarter of the number received at the time when he needs to book the room, but no more than one half.

(c) If Liam is correct:

(i) What is the smallest number of delegates that could be registered for a whole-day course to guarantee that he will make the greatest profit by booking the Large room? [3]

(ii) What is the largest number of delegates that could be registered for a whole-day course to guarantee that he will make the greatest profit by booking the Medium room? [2]

[Turn over for Question 2]

2 *Pumpet* is a game for two players. The first player to win three rounds is the winner.

The game is played with a pack of 36 numbered cards. One single-digit number appears on the face of every card. The numbers from 1 to 9 appear on four cards each.

In each round the players take it in turns to make a total of five claims each. Whichever player is making the current claim is called the 'claimant' and the other player is called the 'judge'. The procedure for each claim is as follows.

- The claimant lays three of the five cards in their hand face down in front of them.
- The claimant then claims a number of points, which must be a multiple of 5.
- The judge then either accepts the claim, if they think that the claim matches the sum of the numbers on the three cards, or challenges it.
- The three cards are then revealed and points are scored as detailed below.

	Claim matches sum	*Claim does not match sum*
Claim accepted	Claimant scores the value of the claim. Judge scores 10 points for judging correctly.	Claimant scores the value of the claim. Judge scores 0.
Claim challenged	Claimant scores double the value of the claim. Judge scores 0.	Claimant scores 0. Judge scores 10 points for judging correctly, plus the value of the sum of the three cards.

At the beginning of each round, both players draw a card from the pack and the player with the higher number (after redrawing if necessary) decides whether to be claimant or judge first. The full pack is then shuffled and placed in front of the players with the cards face down.

Both players take five cards from the top of the pack, without showing them to each other, before the first claim is made. After each claim has been judged and the points recorded, the three cards that were laid down are discarded and the claimant takes the next three cards from the top of the pack (again without showing them to the other player). The winner of the round is the player with the higher score.

(a) What is the greatest total score that one player could possibly have after both players have made their first claim? [2]

Olivia and Gavin are playing a game of *Pumpet*.

In the first round Olivia was the first claimant. Her cards were 9, 9, 6, 5, 1. She claimed 20 points and Gavin accepted her claim. The three cards she had laid down were revealed as 9, 6 and 5. She then drew 9, 9 and 3 from the pack.

Gavin's first claim was 25 points. Olivia had no hesitation in challenging the claim and Gavin's cards were revealed to be 6, 4 and 2.

(b) **(i)** What were Olivia's and Gavin's scores after both had made their first claim? [2]

(ii) How did Olivia know that Gavin's claim did not match the sum of his three cards? [2]

Gavin said later that he was unable to make any multiples of 5 from three of his first five cards, so he had decided to make the lowest sum he could and claim high.

(c) What were the other two of Gavin's first five cards? Explain your reasoning. [3]

Olivia and Gavin have now won two rounds each, so, unless there is a tie, the winner of the fifth round will win the game.

Both players have made four claims in the fifth round and taken three cards from the pack for the final time.

The progress of the round so far is shown in the table below.

Claim number	*Claimant*	*Points claimed*	*Cards*	*Cumulative scores*	
				Olivia	*Gavin*
1	Olivia	15	7, 6, 2	15	10
2	Gavin	20	9, 7, 4	25	30
3	Olivia	20	9, 8, 3	65	30
4	Gavin	15	6, 2, 1	65	45
5	Olivia	20	8, 5, 4	85	45
6	Gavin	15	9, 5, 1	85	75
7	Olivia	15	6, 2, 2	85	95
8	Gavin	15	7, 4, 4	95	110

(d) Which of the claims made so far were challenged? [2]

Olivia's final five cards are 9, 7, 3, 1 and 1. The only claim she can make that would match the sum of three of her cards is for 5 points, which, if accepted by Gavin will put her further behind. Nevertheless, she has worked out that this claim will guarantee that she wins the round and therefore the game.

(e) Explain Olivia's reasoning. [4]

3 Hugh is planning to hold an executive meeting in a boardroom that contains a large circular table surrounded by 25 seats. The table and the seats are fixed to the floor, so if fewer than 25 people attend the meeting then there will be some empty seats around the table.

Hugh does not want there to be any gaps between executives of more than two empty seats.

(a) What is the smallest number of executives (including Hugh) that could attend the meeting without this happening? Explain your answer. [2]

(b) Hugh is considering holding a meeting for 19 executives (including himself). Hugh's wife says, 'With 19 executives and 25 seats, you are certain to have at least one group of at least 4 executives sitting next to each other without a gap.'

Is Hugh's wife correct? Explain your answer. [2]

(c) What is the smallest number of executives (including Hugh) that would need to attend a meeting to be sure of having at least one group of at least 8 executives sitting next to each other without a gap? Explain your answer. [2]

Hugh also wants to arrange a separate meeting, which he will **not** attend, for some of the managers in the company. Another room in the building contains 10 identical circular tables, each surrounded by 12 seats. The managers are instructed to fill up the tables so that the difference between the number of managers on the fullest table and the number on the emptiest table is as small as possible.

(d) What is the smallest number of managers that must attend so that there definitely will not be more than two empty seats next to each other at any table? [1]

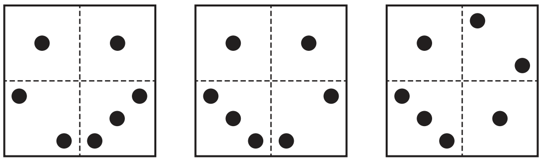
Hugh decides that he will tolerate sometimes having a maximum of three consecutive empty seats, provided that this happens a maximum of twice on fewer than half of the tables in the room and a maximum of once on each of the other tables.

(e) If Hugh creates a seating plan, specifying where each manager must sit, what is the smallest number of managers needed? [3]

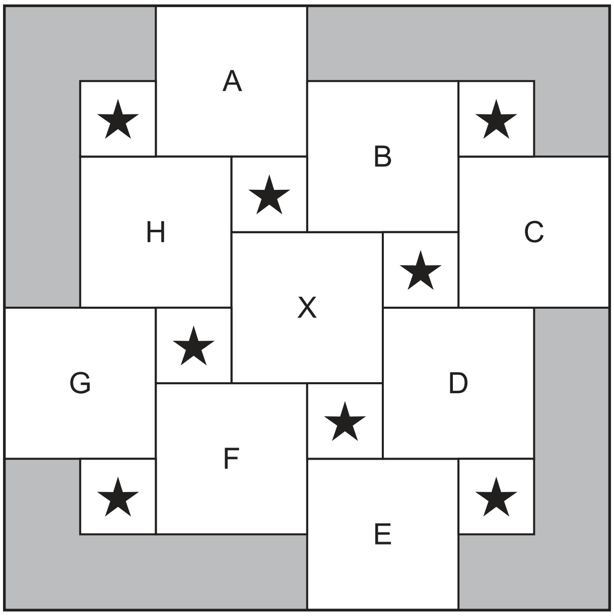
[Turn over for Question 4]

4 *Quadrominoes* is a two-player game that is played with a set of square tiles. Each tile is divided into four sections and has a number of dots between 1 and 3 in each section.

Every tile has at least one section with one dot, at least one section with two dots and at least one section with three dots. There is therefore one number of dots that appears twice on any tile. No two tiles are the same as each other. All of the tiles which have two sections each containing just one dot are shown below.



The board for a game of *Quadrominoes* is shown below.



Each of the large squares is the size of one tile.

To set up the game a tile is chosen at random and placed on the central square (X). The remaining tiles are placed in a bag. Players then take turns alternately. On a turn, the player draws three tiles from the bag, and chooses two to return to the bag. The player then places the remaining tile onto an empty square.

- On the first turn the tile may be placed on any of the empty squares.
- On all other turns the tile cannot be placed so that it is touching the tile that was placed on the previous turn.
- When two tiles touch the number of dots in the adjacent sections must not be equal.

On each turn the player can score points as follows:

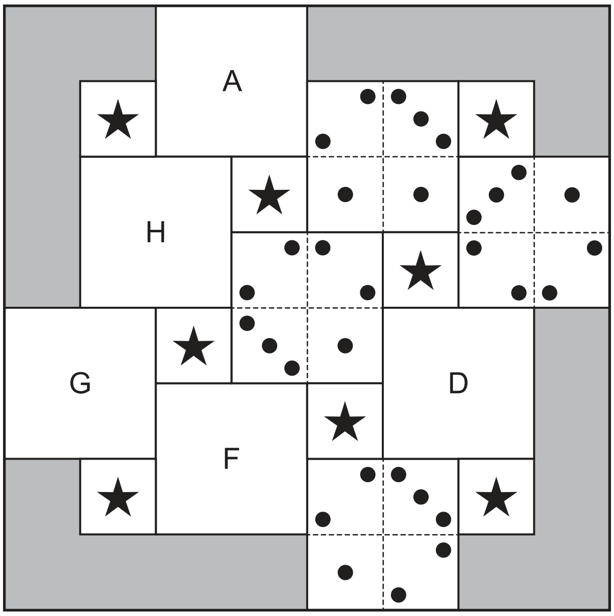
- Each empty square next to the tile just placed scores 1 point.
- If placing the tile causes the total number of dots above, below or to the side of a star to increase from below 6 to above 5, 3 points are scored for each such star.

The game ends when the players have had three turns each. The player with the higher score is the winner.

(a) **(i)** What is the maximum number of points that could be scored on the first turn of the game? [2]

(ii) Explain why at most 10 points can be scored in one turn. [2]

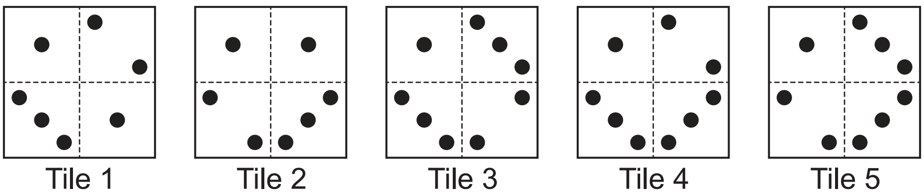
Scott and Miles are playing a game of *Quadrominoes*. Scott played the first tile on square B. The current state of the board is:



(b) **(i)** On which square did Miles place his tile? [1]

(ii) What is the current score in the game? [2]

The five tiles that are left in the bag are shown below.



(c) Assuming that Miles does not place a tile on square F (or square D), which are the two tiles that could not be used to score 6 points for Scott if placed on square D on his next turn? [2]

Miles has drawn tiles 1, 2 and 3 from the bag. As he is not permitted to play on square D, he has decided to play on square G and then play his final turn on square F. He wants to be able to score 9 points when he plays the tile on square F, as that would mean that he would win the game.

Miles assumes that Scott will place a tile in square D on his turn in such a way that only one dot is in the bottom left corner.

(d) **(i)** Explain how Miles can deduce from this that the only way to score 9 points on his final turn would be to place either tile 1 or tile 4 on square F. [3]

(ii) Which tile should Miles play on square G, and in what orientation? Justify your answer. [3]

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